Odredite minimalnu širinu brane a da ne dođe do prevrtanja brane načinjene od betona gustoće $\rho_b = 2900$ kg/m^3 jedinične širine prema slici. Zadano je : $H = 4{,}3$ m, $h = 3{,}4$ m, $\rho = 1000$ kg/m^3.

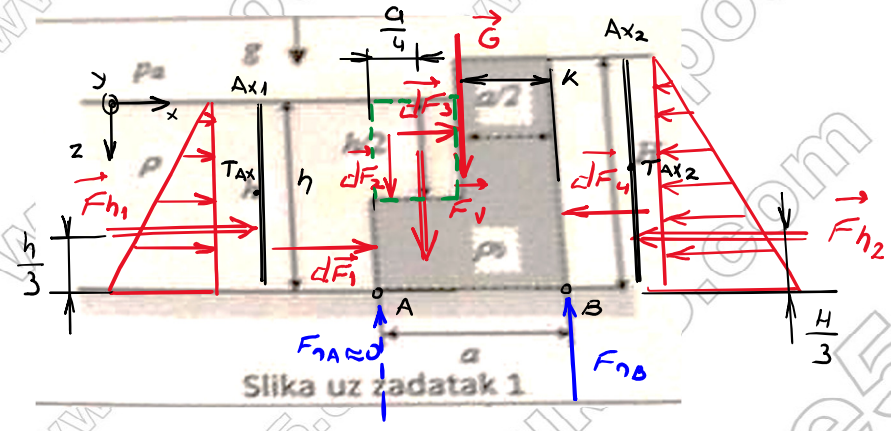
Slika uz zadatak 1

Zadano: $\rho_b = 2900 \frac{kg}{m^3}$, $\rho = 1000 \frac{kg}{m^3}$, $H = 4{,}3\,m$, $h = 3{,}4\,m$

$b = 1\,m$

Traži se: $a = ?$

1. Prikaz sila na branu



Slika uz zadatak 1

Pretpostavka: okretanje će se desiti oko točke B

$$\sum M_B = 0$$

$$-F_{h1}\cdot\frac{h}{3} + F_v\cdot\left(\frac{a}{4}+\frac{a}{2}\right) + G\cdot K + F_{h2}\cdot\frac{H}{3} = 0 \quad (1)$$

1) Sila Fh1

$$F_{h1} = p_{mTAx_1}\cdot A_{x_1}\ [N] \quad (2)$$

$$p_{mTAx_1} = s\cdot g\cdot z_{TAx_1}$$

$$z_{TAx1} = \frac{h}{2}$$

$$A_{x_1} = h\cdot b$$

$$F_{h1} = \frac{1}{2}\,s\,g\,b\,h^2\ [N] \quad (2.1)$$

2) Sila Fh2

$$F_{h2} = p_{mTAx_2}\cdot A_{x_2}\ [N] \quad (3)$$

$$p_{mTAx_2} = s\cdot g\cdot z_{TAx_2}\ [Pa]$$

$$z_{TAx_2} = \frac{H}{2}$$

$$A_{x_2} = b\cdot H\ [m^2]$$

$$F_{h2} = \frac{1}{2}\,s\,g\,b\,H^2\ [N] \quad (3.1)$$

3) Sila Fv

$$F_V = S \cdot g \cdot V_s \quad [N] \quad (4)$$

$$V_s = A_s \cdot b \quad [m^3]$$

$$A_s = \frac{h}{2} \cdot \frac{a}{2}$$

$$F_V = \frac{1}{4} \rho g a \cdot h \cdot b \quad [N] \quad (4.1)$$

4) Sila Zemljine teže, G

$$G = M \cdot g \quad [N] \quad (5)$$

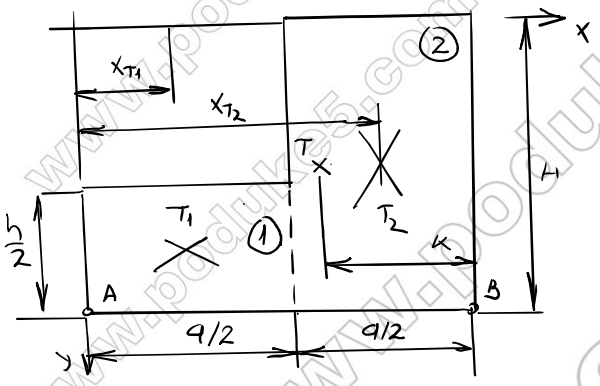
$$M = \rho_b \cdot V_b \quad [kg]$$

$$V_b = \left[\left(H \cdot \frac{a}{2}\right) + \left(\frac{h}{2} \cdot \frac{a}{2}\right)\right] b \quad [m^3]$$

$$G = \rho_b \cdot g \cdot \frac{a}{2} \cdot b \left(H + \frac{h}{2}\right) \quad (5.1)$$

5) Krak k

Težište površine



$$A \cdot x_T = A_1 x_{T_1} + A_2 x_{T_2} \quad (6)$$

$$A_1 = \frac{a}{2} \cdot \frac{h}{2}$$

$$A_2 = \frac{a}{2} H$$

$$x_{T_1} = \frac{1}{2} \cdot \frac{a}{2} = \frac{1}{4} a$$

$$x_{T_2} = \frac{a}{2} + \frac{1}{2} \frac{a}{2} = \frac{3}{4} a$$

$$A = A_1 + A_2$$

$$x_T = \frac{\frac{a \cdot h}{4} \cdot \frac{a}{4} + \frac{1}{2}(aH) \cdot \frac{3}{4} a}{\frac{a \cdot h}{4} + \frac{aH}{2}}$$

$$x_T = \frac{\frac{a^2 \cdot 3,4}{16} + \frac{3 a^2 \cdot 4,3}{8}}{a\left(\frac{3,4}{4} + \frac{4,3}{2}\right)}$$

$$x_T = 0,60833\, a$$

Sa skice: $K = a - x_T = a - 0,60833\, a$

$$K = 0,39167\, a \quad (7)$$

Iz (1): $$-\frac{1}{2} \rho_b g b \frac{h^3}{3} + \frac{1}{4} \rho g a h b \cdot \frac{3}{4} a + \rho_b \cdot g \cdot \frac{a}{2} \cdot b \cdot \left(H + \frac{h}{2}\right) \cdot 0,39167 a + \frac{1}{2} \rho g b H^2 \cdot \frac{H}{3} = 0$$

$$-\frac{1}{2} \cdot 1000 \cdot 9,81 \cdot 1 \cdot \frac{3,4^3}{3} + \frac{3}{16} \cdot 1000 \cdot a^2 \cdot 9,81 \cdot 3,4 \cdot 1 + 2700 \cdot 9,81 \cdot 0,39167 \cdot \frac{a^2 \cdot 1}{2} \cdot \left(4,3 + \frac{3,4}{2}\right) + \frac{1}{2} \cdot 1000 \cdot 9,81 \cdot 1 \cdot \frac{4,3^3}{3} = 0$$

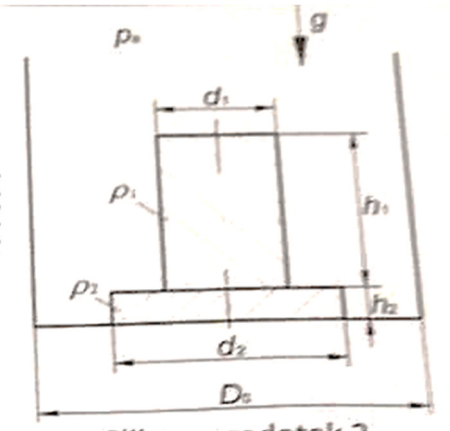
$$39\,681{,}45\,a^2 = -97\,862{,}93$$

Ova jednadžba pokazuje da je pogrešno pretpostavljena točka vrtnje, brana se ne bi okretala oko točke B nego oko točke A

$$a = \sqrt{\frac{97\,862{,}93}{39\,681{,}45}}$$

$$a = 1{,}5704\ m$$

2.
U cilindričnoj posudi prema slici, nalazi se predmet sastavljen od 2 valjka različite gustoće. Odredite volumen V vode gustoće ρ, koji treba uliti u posudu da bi predmet počeo plivati. Zadano je: $D_0 = 54{,}3$ cm, $d_1 = 28{,}5$ cm, $h_1 = 22{,}6$ cm, $d_2 = 40$ cm, $h_2 = 1$ cm, $\rho_1 = 320$ kg/m^3, $\rho_2 = 7800$ kg/m^3, $\rho = 998$ kg/m^3.



Slika uz zadatak 2

Da bi se predmet odvojio od posude, potrebno je da sila uzgona

Postane veća od težine predmeta:

$$F_U \geq F_g$$

1. Sila uzgona, Fu

$$F_U = S \cdot g \cdot V_i \quad [N] \quad (1)$$

$$V_i = V_1 + V_2 \quad [m^3] \quad (2)$$

$$V_1 = \frac{d_1^2 \pi}{4} h_1 \quad (3)$$

$$V_2 = \frac{d_2^2 \pi}{4} h_2 \quad (4)$$

2. Sila Zemljine teže, Fg

$$F_g = F_{g1} + F_{g2} \quad [N] \quad (5)$$

a) $$F_{g1} = m_1 \cdot g \quad [N] \quad (6)$$

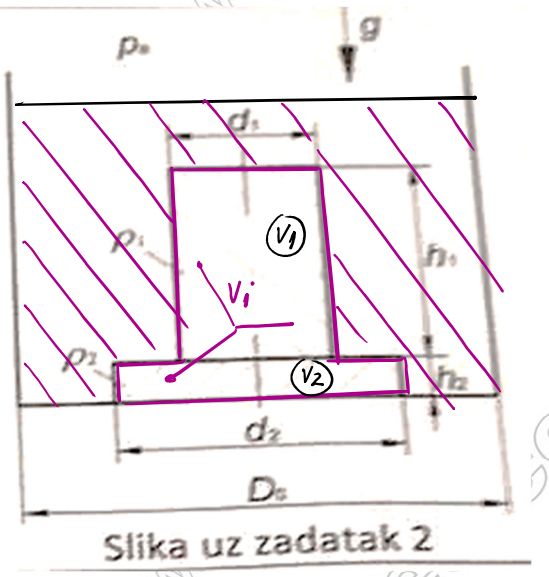
$$m_1 = S_1 V_1 \quad [kg] \quad (7)$$

$$F_{g1} = S_1 \frac{d_1^2 \pi}{4} h_1 \cdot g \quad [N] \quad (6.1)$$

b) $$F_{g2} = m_2 \cdot g \quad [N] \quad (8)$$

$$m_2 = S_2 V_2 \quad [kg] \quad (9)$$

$$F_{g2} = S_2 \frac{d_2^2 \pi}{4} h_2 \cdot g \quad [N] \quad (8.1)$$



Slika uz zadatak 2

Iz (1): $$V_i \geq \frac{S_1 \frac{d_1^2 \pi}{4} h_1 \cdot g + S_2 \frac{d_2^2 \pi}{4} h_2 g}{S g}$$

$$V_i \geq \frac{320 \cdot 0{,}285^2 \cdot 0{,}226 + 7800 \cdot 0{,}4^2 \cdot 0{,}01}{998} \cdot \frac{\pi}{4}$$

$$V_i \geq 0{,}014\,444\ m^3$$

$$V_i \approx 14{,}4\ l$$

Ukupni obujam tijela:

$$V = V_1 + V_2 = \frac{d_1^2 \pi}{4} h_1 + \frac{d_2^2 \pi}{4} h_2$$

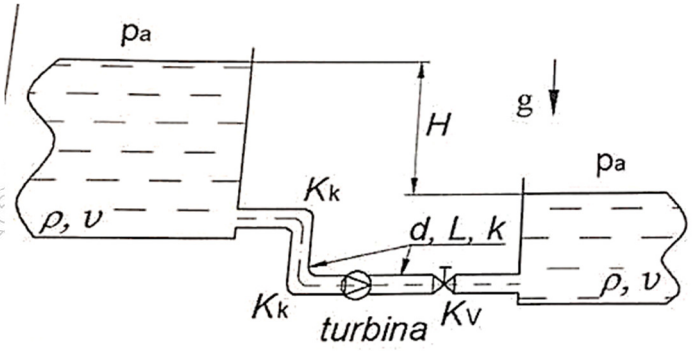
$$V = (0{,}285^2 \cdot 0{,}226 + 0{,}4^2 \cdot 0{,}01) \cdot \frac{\pi}{4}$$

$$V = 0{,}015\,674\ m^3$$

Potrebno je uliti 14,5 l vode da se tijelo počne odvajati od posude.

3.

Odredite snagu P_T koju ostvaruje turbina ugrađena u sustav prema slici ako je na turbini izmjeren neto pad od $h_T = 19$ m. Zadano je $d = 0{,}16$ m, $k = 0{,}015$ mm, $L = 462$ m, $H = 26$ m, $\rho = 998{,}2$ kg/m^3, $\nu = 1{,}52 \cdot 10^{-6}$ m^2/s, $K_k = 0{,}8$, $K_V = 23$.



1. Snaga turbine, PT

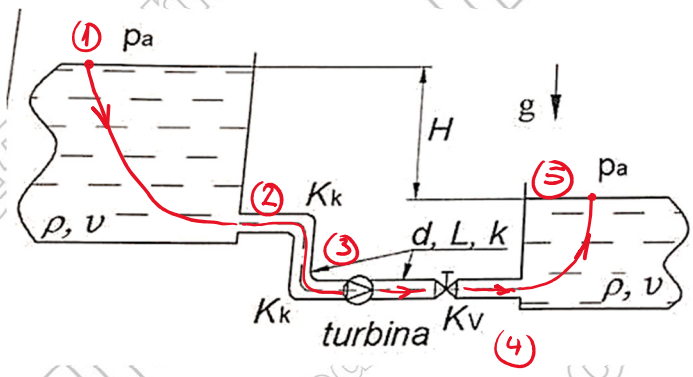
$$P_T = \eta_T \cdot P_{Tt} \ [W] \quad (1)$$

Teorijska snaga turbine, PTt

$$P_{Tt} = \dot{m} \cdot g \cdot h_T = \rho \cdot Q_V \cdot g \cdot h_T \ [W] \quad (2)$$

2. Protok kroz cjevovod

Inženjerska Bernoullieva jednadžba za presjeke 1 – 5



$$\alpha_1 \frac{v_{1S}^2}{2g} + \frac{p_1}{\rho g} + z_1 = \alpha_5 \frac{v_{5S}^2}{2g} + \frac{p_5}{\rho g} + z_5 + h_T + h_g \ [m] \quad (3)$$

1) $v_{1S} \approx 0 \ \frac{m}{s}$ (PVS)

2) $p_1 = p_a$

3) $z_1 - z_5 = H$

4) $v_{5S} \approx 0 \ \frac{m}{s}$

5) $p_5 = p_a$

$$H = h_T + h_g \quad (3.1)$$

1) Gubici u cjevovodu, hg

$$h_g = h_{gli} + h_{glok} \ [m] \quad (4)$$

Linijski gubici u cjevovodu

$$h_{gli} = \lambda \frac{v_{sc}^2}{2g} \cdot \frac{L}{d} \ [m] \quad (5)$$

Mjesni gubici u cjevovodu

$$h_{glok} = h_{gi2} + h_{gK1} + h_{gK2} + h_{gV} + h_{gul} \ [m] \quad (6)$$

$$h_{gi2} \approx 0{,}5 \frac{v_{sc}^2}{2g} \ [m] \quad (7)$$

$$h_{gK1} = h_{gK2} = k_K \cdot \frac{v_{sc}^2}{2g} \ [m] \quad (8)$$

$$h_{gV} = k_V \cdot \frac{v_{sc}^2}{2g} \ [m] \quad (9)$$

$$h_{gul} = 1 \cdot \frac{v_{sc}^2}{2g}$$

$$h_{glok} = (0{,}5 + 2 \cdot 0{,}8 + 23 + 1) \frac{v_{sc}^2}{2g}$$

$$h_{glok} \approx 26{,}1 \cdot \frac{v_{sc}^2}{2g}$$

Sada je (3.1): $H = h_T + \lambda \frac{v_{sc}^2}{2g} \frac{L}{d} + 26{,}1 \frac{v_{sc}^2}{2g}$ (3.2) $\lambda = ?$ $v_{sc} = ?$

$$26 - 19 = \lambda \frac{v_{sc}^2}{2 \cdot 9{,}81} \cdot \frac{462}{0{,}16} + \frac{26{,}1}{2 \cdot 9{,}81} \cdot v_{sc}^2$$

$$7 = 1{,}330\, v_{sc}^2 + 147{,}171\, \lambda\, v_{sc}^2$$

$$v_{sc} = \sqrt{\frac{7}{1{,}33 + 147{,}171\,\lambda}}$$

Pogađanje rezultata

Relativna hrapavost cijevi $e = \frac{k}{d} = \frac{0{,}015\ mm}{160\ mm} = 9{,}375 \cdot 10^{-5}$

1) $\lambda_p = 0{,}015$

$$v_{sc} = \sqrt{\frac{7}{1{,}33 + 147{,}171 \cdot 0{,}015}}$$

$$v_{sc} = 1{,}407\ \frac{m}{s}$$

a) $Re = \frac{v_{sc} \cdot d}{\nu} = \frac{1{,}407 \cdot 160}{1{,}52 \cdot 10^{-6}}$

$Re = 148\,105\,263{,}2$

b) $\lambda_i = \frac{1{,}325}{\left[\ln\left(\frac{0{,}015}{3{,}7 \cdot 160} + \frac{5{,}74}{(148\,105\,263{,}2)^{0{,}9}}\right)\right]^2}$ $\qquad \lambda = \frac{1,325}{\left[\ln\left(\frac{k}{3,7D} + \frac{5,74}{Re^{0,9}}\right)\right]^2}$

$\lambda_i = 0{,}012 \neq \lambda_p = 0{,}015$

2) $\lambda_p = 0{,}012$

a) $v_{sc} = \sqrt{\frac{7}{1{,}33 + 147{,}171 \cdot 0{,}012}}$

$v_{sc} = 1{,}504\ \frac{m}{s}$

b) $Re = \frac{1{,}504 \cdot 160}{1{,}52 \cdot 10^{-6}} = 1\,582\,782\,68{,}8$

c) $\lambda_i = \frac{1{,}325}{\left[\ln\left(\frac{0{,}015}{3{,}7 \cdot 160} + \frac{5{,}74}{1\,582\,782\,68{,}8^{0{,}9}}\right)\right]^2}$

$\lambda_i = 0{,}011\,85 \approx \lambda_p = 0{,}012 \Rightarrow v_{sc} = 1{,}504\ \frac{m}{s}$

Iz (1) i (2):

$$P_T = P_{T_t} = S\, v_{sc}\, \frac{d^2 \pi}{4} \cdot g \cdot h_T \quad [W]$$

$$P_T = 998{,}2 \cdot 1{,}504 \cdot \frac{0{,}16^2 \pi}{4} \cdot 9{,}81 \cdot 19$$

$P_T = 5626{,}23\ W$